

Maxi-sizing the trilinear Higgs self-coupling: how large could it be?

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ABSTRACT: In order to answer the question on how much the trilinear Higgs self-coupling could deviate from its Standard Model value in weakly coupled models, we study both theoretical and phenomenological constraints. As a first step, we discuss this question by modifying the Standard Model using effective operators. Considering constraints from vacuum stability and perturbativity, we show that only the latter can be reliably assessed in a model-independent way. We then focus on UV models which receive constraints from Higgs coupling measurements, electroweak precision tests, vacuum stability and perturbativity. We find that the interplay of current measurements with perturbativity already exclude self-coupling modifications above a factor of few with respect to the Standard Model value.

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1 Introduction

The recent discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2] marks a milestone event for high-energy physics. Yet, the Higgs boson is only a remnant of the underlying mechanism of spontaneous electroweak (EW) symmetry breaking, the so-called Brout-Englert-Higgs mechanism [3, 4]. In order to improve our understanding of the dynamics initiating EW symmetry breaking, a key ingredient is the global structure of the scalar potential that triggers the spontaneous breaking of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{QED}}$. While the ongoing LHC program, focusing on precise measurements of Higgs and gauge boson masses and couplings, will continue to improve our understanding of the potential's local structure

in the vicinity of the EW minimum, information on the shape of the vacuum in a model-independent way is experimentally very difficult to obtain.*

However, if one specifies the degrees of freedom and interactions in the scalar sector, one can calculate the form of the scalar potential. After EW symmetry breaking such potential gives rise to multi-scalar interactions, i.e. at lowest order cubic and quartic Higgs self-interactions. While the former can be probed directly in searches for multi-Higgs final states [7–27], indirectly via their effect on precision observables [28, 29] or loop corrections to single Higgs production [30–34], the latter are inaccessible at the LHC or a future linear collider [35–37]. Thus, to obtain a glimpse at the shape of the scalar potential we have to focus on the cubic scalar self-coupling.

If new light degrees of freedom contribute to the Higgs potential, they typically dominate the multi-Higgs phenomenology. On the other hand, if new degrees of freedom are heavy, it is widely argued that the Effective Field Theory (EFT) approach is most suitable to study deformations of the Standard Model (SM) Higgs potential in a rather model-independent and predictive way. Thus, in the latter case, where we assume that no light states below the cutoff scale $\Lambda \gg v \equiv 246$ GeV exist, it is tempting to introduce an operator $|H|^6$ and connect the (global) properties of the vacuum, e.g. whether the EW minimum is a local or global one, with the cubic Higgs self-coupling. In particular, one could consider using vacuum stability arguments to infer model-independent bounds on the triple Higgs coupling.

In this work, we show that this approach is flawed. In particular, there can be two kinds of instabilities corresponding to the possible emergence of new minima either at large field values $v \ll H \lesssim \Lambda$ or at $H = 0$. The former, is shown to be spurious since the very expansion of the scalar potential in powers of H/Λ in the vicinity of an instability leads to the breakdown of the EFT expansion [38]. In Sect. 2 we explicitly show that a weakly coupled toy model can feature an absolutely stable vacuum in the full theory, while obtaining a spurious instability in the EFT limit. Similarly, the second type of instability, due to the emergence of a new minimum in $H = 0$, is also shown to be not under control when including only the lowest terms in the EFT expansion.

On the other hand, allowing for too large Higgs self-couplings (either trilinear or quadrilinear ones) raises the question of the validity of perturbative methods. When tree-level scattering amplitudes violate unitarity, large higher-order corrections are necessary to restore unitarity, thus leading to the breakdown of the perturbative expansion. This argument has been employed in the past to set theoretical bounds on couplings and scales. The most famous example is the scattering of longitudinal vector bosons, which has been used to set a theoretical limit on the Higgs boson mass by performing a partial wave analysis [39]. We apply this method in Sect. 2.3 in order to set a bound on Higgs self-couplings by considering the $hh \rightarrow hh$ scattering. In addition, we show that the requirement that the loop-corrected Higgs scalar vertices are smaller than their tree-level values gives a very similar theoretical

*The energy scale of non-perturbative phenomena, e.g. the mass of the $SU(2)_L$ sphalerons [5], could potentially allow to probe the scalar potential away from the EW minimum [6].

bound on Higgs self-couplings.

Given the apparent limitations of the EFT framework in setting bounds beyond perturbativity, we focus on UV complete scenarios from Sect. 3 onwards to investigate the question of the maximally allowed triple Higgs coupling. We consider for simplicity only weakly coupled models, as they retain a higher degree of predictivity and we have full control of the theory. Particularly large deviations are expected in scenarios where the SM is augmented by extra scalars. We focus on new scalars Φ which can couple via a tadpole operator of the type $\mathcal{O}_\Phi = \Phi f(H)$, where $f(H)$ is a string of Higgs fields (or their charge conjugates). In Sect. 3 we argue that such couplings potentially give the largest contributions to the Higgs self-coupling and classify all the possible representations of Φ that lead to such interactions. As a result of the presence of the new scalars, the vacuum structure of the scalar potential is more contrived and it becomes challenging to establish a direct relation between Higgs self-coupling deviations and the stability of the EW vacuum. Still, parts of the parameter space can be excluded by requiring the vacuum to be (meta)stable. In addition, we take into account phenomenological limits from Higgs coupling measurements and EW precision tests. Together with a perturbativity requirement for the parameters of the extended scalar potential, we find that maximal deviations up to few times the SM trilinear Higgs self-coupling are still feasible.

Looking beyond tree level, we investigate loop-induced modifications in Sect. 3.3. While such contributions are expected to be smaller, they are of particular interest as they are induced by a plethora of new physics models. We discuss here the case of fermionic loops, since in such a case one can regain a direct correlation between the triple Higgs coupling and the stability of the EW vacuum. We comment on this relation, explicitly studying the case of low-scale seesaw models, which are largely unconstrained by other Higgs couplings' measurements. Finally, in Sect. 4 we present our conclusions.

2 Theoretical constraints on Higgs self-couplings

Let us parametrize the Higgs potential in the SM broken phase as

$$V(h) = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\lambda_{hhh}h^3 + \frac{1}{4!}\lambda_{hhhh}h^4, \quad (2.1)$$

where λ_{hhh} (λ_{hhhh}) denotes the modified trilinear (quadrilinear) Higgs self-coupling. In the SM we have

$$\lambda_{hhh}^{\text{SM}} = \frac{3m_h^2}{v} \simeq 190 \text{ GeV} \quad \text{and} \quad \lambda_{hhhh}^{\text{SM}} = \frac{3m_h^2}{v^2} \simeq 0.77. \quad (2.2)$$

The question we want to address is whether there exist some model-independent bounds on the value of the Higgs self-couplings. To this end, we will consider two classes of theoretical constraints which are vacuum stability and perturbativity. While the latter is, strictly speaking, not a bound, it is still interesting given our limitations in using the Lagrangian in Eq. (2.1) beyond perturbation theory. In Sect. 2.3 we will provide a simple perturbativity criterion which can be applied to the potential of Eq. (2.1). In order to formulate the question

in a gauge invariant way we will add an operator $\frac{c_6}{v^2} |H|^6$ to the SM Lagrangian and study the vacuum structure of the theory. Is it then possible to set model-independent bounds on the Wilson coefficient c_6 from the requirement that the EW vacuum is absolutely stable or long-lived enough? As we are going to see, the answer to the latter question is in general negative, requiring a careful analysis of the range of applicability of the EFT.

2.1 EW symmetry breaking with $d = 6$ operators

We start by reviewing EW symmetry breaking in the SM augmented by the operator $|H|^6$ (see e.g. [40]). The truncated potential reads

$$V^{(6)}(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{c_6}{v^2} |H|^6, \quad (2.3)$$

where the normalization of the $d = 6$ operator is given in terms of $v = (\sqrt{2}G_\mu)^{-1/2} \simeq 246$ GeV. Note that $c_6 = \bar{c}_6 \lambda$ in the notation of Ref. [41]. In the following, we will focus on weakly coupled regimes, where c_6 is at most of $\mathcal{O}(v^2/\Lambda^2)$ and Λ is the cutoff of the EFT.[†]

In order to minimize the potential, we project the Higgs doublet on its background real component, $H \rightarrow \frac{1}{\sqrt{2}} \bar{h}$. From the equation

$$V^{(6)'}(\bar{h}) = \left(-\mu^2 + \lambda \bar{h}^2 + \frac{3c_6}{4v^2} \bar{h}^4 \right) \bar{h} = 0, \quad (2.4)$$

we find three possible stationary points: $\bar{h} = 0$, v_+ , v_- , where

$$v_\pm^2 = \frac{2v^2}{3c_6} \left(-\lambda \pm |\lambda| \sqrt{1 + \frac{3c_6 \mu^2}{\lambda^2 v^2}} \right) = (\pm |\lambda| - \lambda) \frac{2v^2}{3c_6} \pm \frac{\mu^2}{|\lambda|} \mp \frac{3c_6 \mu^4}{4|\lambda|^3 v^2} + \mathcal{O}(c_6^2), \quad (2.5)$$

and in the last step we expanded for $c_6 \ll 1$. The nature of the stationary points (whether they correspond to maxima or minima) depends on the second derivative of the potential

$$V^{(6)''}(\bar{h}) = -\mu^2 + 3\lambda \bar{h}^2 + \frac{15c_6}{4v^2} \bar{h}^4. \quad (2.6)$$

Considering the possible signs of the potential parameters in Eq. (2.3) we have in total $2^3 = 8$ combinations, out of which only 4 lead to a phenomenologically viable (i.e. $\bar{h} \neq 0$) EW minimum:

1. $\mu^2 > 0$, $\lambda > 0$, $c_6 > 0$: In this case Eq. (2.5) yields (neglecting $\mathcal{O}(c_6^2)$ terms)

$$v_+^2 \simeq \frac{\mu^2}{\lambda} \left(1 - \frac{3c_6 \mu^2}{4\lambda^2 v^2} \right), \quad (2.7)$$

$$v_-^2 \simeq -\frac{4\lambda v^2}{3c_6} \left(1 + \frac{3c_6 \mu^2}{4\lambda^2 v^2} \right). \quad (2.8)$$

[†]By naive dimensional analysis the scaling of c_6 is $g_*^4 v^2/\Lambda^2$, where g_* denotes a generic coupling which can range up to 4π in strongly-coupled theories (see e.g. [42]). However, in theories where the Higgs mass is protected by an additional symmetry, like e.g. in composite Higgs models, the scaling of the coefficient c_6 is expected to be $c_6 \sim \lambda g_*^2 v^2/\Lambda^2 = \lambda v^2/f^2$, with $1/f \equiv g_*/\Lambda$ [41, 43]. Hence, also in this case values of $c_6 \sim 1$ lead to the breakdown of the EFT expansion.

As $c_6 > 0$, only v_+ is a stationary point and from Eq. (2.6) we find

$$V^{(6)''}(0) = -\mu^2 < 0, \quad (2.9)$$

$$V^{(6)''}(v_+) \simeq 2\mu^2 \left(1 + \frac{3c_6\mu^2}{4\lambda^2 v^2} \right) > 0. \quad (2.10)$$

Hence, $\bar{h} = 0$ is a maximum, while $\bar{h} = v_+$ can be identified with the EW minimum v . Note that in the $c_6 \rightarrow 0$ limit we recover the SM result.

2. $\mu^2 > 0, \lambda > 0, c_6 < 0$: In addition to $\bar{h} = 0$ and v_+ , as before, we have a third stationary point v_- , as now $c_6 < 0$ (cf. Eq. (2.8)). The latter corresponds to a maximum, as implied by

$$V^{(6)''}(v_-) \simeq \frac{8\lambda^2 v^2}{3c_6} \left(1 + \frac{9c_6\mu^2}{4\lambda^2 v^2} \right) < 0. \quad (2.11)$$

The potential, which is sketched in the left panel of Fig. 1, features an instability at large field values $\bar{h} \gtrsim v_- \sim \sqrt{\lambda}\Lambda$ (where we used $c_6 \sim v^2/\Lambda^2$). The instability looks however specious, because it is close to the cutoff of the EFT. As in the previous case, for $c_6 \rightarrow 0$ we recover the SM since the position of the second maximum is pushed to infinity.

3. $\mu^2 < 0, \lambda < 0, c_6 > 0$: Substituting in Eq. (2.5) we get

$$v_+^2 \simeq \frac{4|\lambda|v^2}{3c_6} \left(1 + \frac{3c_6\mu^2}{4\lambda^2 v^2} \right), \quad (2.12)$$

$$v_-^2 \simeq -\frac{\mu^2}{|\lambda|} \left(1 - \frac{3c_6\mu^2}{4\lambda^2 v^2} \right), \quad (2.13)$$

while the second derivatives of the potential read

$$V^{(6)''}(0) = -\mu^2 > 0, \quad (2.14)$$

$$V^{(6)''}(v_+) \simeq \frac{8\lambda^2 v^2}{3c_6} \left(1 + \frac{9c_6\mu^2}{4\lambda^2 v^2} \right) > 0, \quad (2.15)$$

$$V^{(6)''}(v_-) \simeq 2\mu^2 \left(1 + \frac{3c_6\mu^2}{4\lambda^2 v^2} \right) < 0. \quad (2.16)$$

Thus $\bar{h} = 0$ and v_+ are minima, while v_- is a maximum. Note that the potential is upturned with respect to that of case 2. (cf. solid curve in the right panel of Fig. 1). This time, however, we must identify the EW minimum v with $v_+ \sim \sqrt{|\lambda|}\Lambda$ (where we used $c_6 \sim v^2/\Lambda^2$), which means that the EW vacuum expectation value (vev) is generated by the physics at the cutoff scale. This corresponds to a non-decoupling EFT, since in the $c_6 \rightarrow 0$ limit the EW minimum is pushed to infinity and we do not re-obtain the SM.

4. $\mu^2 > 0$, $\lambda < 0$, $c_6 > 0$: This case is similar to the previous one, with the difference that $\bar{h} = 0$ is a maximum (cf. Eq. (2.14)), the maximum in v_- disappears (cf. Eq. (2.13)), while the Λ dominated EW minimum remains in v_+ (cf. Eq. (2.15)). Also in this case the limit $c_6 \rightarrow 0$ does not reproduce the SM.

2.2 Vacuum instabilities

There are essentially two types of instabilities associated with the presence of the coupling c_6 : the most obvious one, at large field values, is triggered by a negative c_6 (case 2 in Sect. 2.1), while the other one has to do with the destabilization of the EW ground state against the minimum in $\bar{h} = 0$ (case 3 in Sect. 2.1), which happens for large enough (positive) values of c_6 (see also the right plot in Fig. 1 shown as a dashed line). This might suggest that there

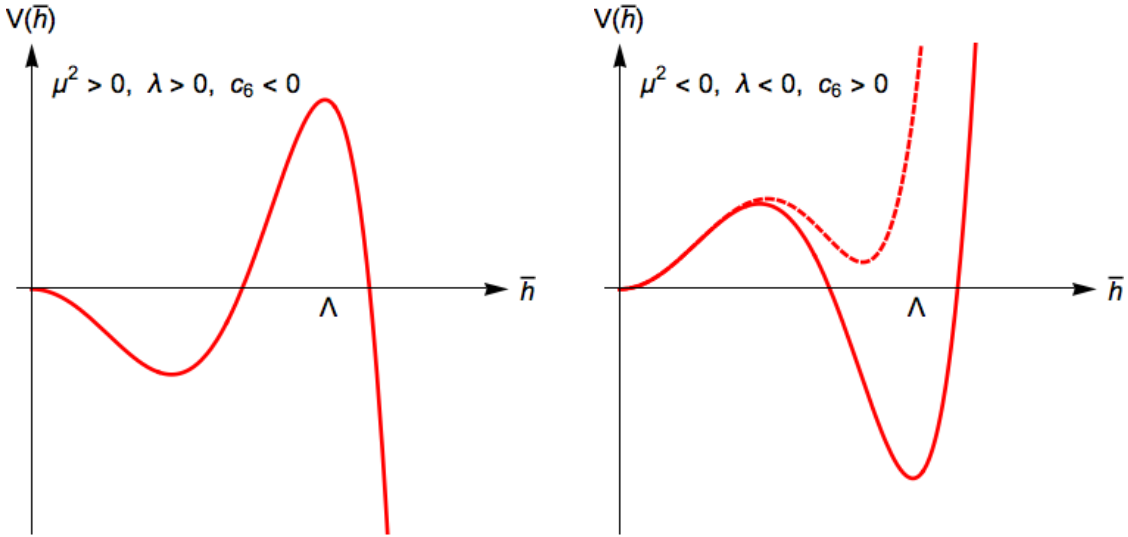


Figure 1. The two kind of instabilities triggered by a sizable c_6 . *Left:* A negative c_6 is responsible for a large-field-value instability close to the scale Λ . *Right:* The EW minimum is generated by the physics at the cutoff scale Λ . For large enough $c_6 > 0$, the absolute minimum is in $\bar{h} = 0$ (dashed line), and the EW vacuum gets destabilized.

is a lower and upper bound on c_6 by the requirement that the EW minimum is the absolute one. However, we are going to argue that there is no such a model-independent bound within a generic EFT. Let us discuss in turn the two kind of instabilities.

2.2.1 Large-field-value instability: $\bar{h} \lesssim \Lambda$

The main observation here is that the very expansion of the scalar potential in powers of \bar{h}/Λ in the vicinity of an instability leads to the breakdown of the EFT expansion [38].[‡]

[‡]This instability was discussed in a slightly different context in Ref. [38]. There it was shown that the effect of an $|H|^6$ operator on the vacuum stability analysis of the SM is always small, whenever it can be reliably computed within the EFT.

This has to be traced back to the fact that when the scalar potential is close to vanish, field configurations $\bar{h} \sim \Lambda$ do not cost prohibitive energy to excite, contrary to the standard case $V(\bar{h} \sim \Lambda) \sim \Lambda^4$.

The spurious nature of the $|H|^6$ instability is clearly exemplified by taking the EFT limit of a simple toy model that features, by construction, absolute stability in the full theory [38]. Let h and ϕ be two real scalar fields, whose potential reads

$$V(h, \phi) = -\frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}M^2\phi^2 + \xi h^3\phi + \kappa h^2\phi^2 + \frac{1}{4}\lambda'\phi^4. \quad (2.17)$$

We note that the structure of this potential closely resembles that of a Higgs doublet coupled to an EW quadruplet with hypercharge $Y = 3/2$. Let us consider now the limit $M^2 \gg m^2 > 0$. The stationary equations can be solved perturbatively for $m^2/M^2 \ll 1$, thus yielding

$$\langle h \rangle \simeq \left(\frac{m^2}{\lambda} \right)^{\frac{1}{2}}, \quad (2.18)$$

$$\langle \phi \rangle \simeq -\frac{\xi}{M^2} \left(\frac{m^2}{\lambda} \right)^{\frac{3}{2}} \ll \langle h \rangle, \quad (2.19)$$

which is a global minimum as long as $M^2 > \frac{9\xi^2}{2\lambda^2}m^2$. Moreover, a sufficient condition for the potential to be bounded from below is

$$\kappa > 0, \quad \wedge \quad \lambda > \frac{\xi^2}{\kappa}, \quad \wedge \quad \lambda' > 0, \quad (2.20)$$

so by choosing the potential parameters as in Eq. (2.20) it is always possible to ensure that the vacuum in Eqs. (2.18)–(2.19) is absolutely stable.

Now we integrate Φ out. A standard calculation yields

$$V_{\text{EFT}}(h) \simeq -\frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 - \frac{1}{2}\xi^2 \frac{h^6}{M^2 + 2\kappa h^2}. \quad (2.21)$$

As a consequence of Eq. (2.20) the EFT potential in Eq. (2.21) is clearly stable as well. On the other hand, by expanding the denominator of the h^6 term for $M^2 \gg 2\kappa h^2$, we get

$$V_{\text{EFT}}(h) \simeq -\frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 - \frac{1}{2}\frac{\xi^2}{M^2}h^6 + \frac{\xi^2\kappa}{M^4}h^8 + \dots \quad (2.22)$$

Apparently, the h^6 operator features an instability, which however is not supported by the full renormalizable model in view of the stability conditions in Eq. (2.20). The key point is that the spurious instability sourced by the h^6 term does not capture the κ dependence, as the appropriate resummation of the geometric series shows in Eq. (2.21). Hence, we conclude that it is not possible to set a model-independent bound on c_6 from the requirement of stability at large field values.

2.2.2 Low-scale instability: $\bar{h} = 0$

In order to study this case it is more convenient to trade the parameters μ^2 and λ in terms of the EW vev v and the physical Higgs mass m_h . Imposing the existence of the EW minimum $\bar{h} = v$ from Eq. (2.4) and expanding over the Higgs field fluctuations $v \rightarrow v + h$, one gets

$$\mu^2 = \lambda v^2 + \frac{3}{4}c_6 v^2 = \frac{m_h^2}{2} - \frac{3}{4}c_6 v^2, \quad (2.23)$$

$$\lambda = \frac{m_h^2}{2v^2} - \frac{3}{2}c_6. \quad (2.24)$$

By substituting $v = 246$ GeV and $m_h = 125$ GeV in Eqs. (2.23)–(2.24), we find $\mu^2 < 0$ and $\lambda < 0$ as long as $c_6 \gtrsim 0.17$. This is precisely the situation described in case 3 of Sect. 2.1. By taking an even larger c_6 it can happen that the minimum in $\bar{h} = 0$ becomes the absolute one (cf. Fig. 1). This happens for (see also [32])

$$V^{(6)}(v) = \frac{c_6 v^4 - m_h^2 v^2}{8} > 0 = V^{(6)}(\bar{h} = 0), \quad (2.25)$$

corresponding to $c_6 \gtrsim 0.26$. However, for a weakly coupled theory where c_6 scales like v^2/Λ^2 , such value of c_6 implies a very low cutoff scale of $\Lambda \lesssim 480$ GeV, thus making the application of the EFT questionable. Even admitting for a strongly-coupled origin of c_6 , the main point here is that higher-order operators cannot be consistently neglected for assessing the global structure of the Higgs potential away from the EW minimum, since $|H|^6$ can give access only up to the sixth derivative of the potential on the EW minimum.

2.3 Perturbativity bounds

On general grounds, one expects that too large values of the Higgs self-couplings are bounded by perturbativity arguments. In the following, we compare two criteria: the first is based on the partial-wave unitarity of the Higgs bosons' scattering amplitude, while the second one consists in the requirement that the loop corrections to the Higgs self-interaction vertices are smaller than the tree-level ones. Both criteria yield a similar result.

2.3.1 Partial-wave unitarity

The $2 \rightarrow 2$ Higgs bosons' scattering amplitude grows indefinitely for large values of the Higgs self-couplings, leading to unitarity violation and hence to the breakdown of the perturbative expansion. Using the modified Lagrangian in Eq. (2.1), the $hh \rightarrow hh$ scattering amplitude reads (see also Fig. 2).

$$\mathcal{M} = -\lambda_{hhh}^2 \left(\frac{1}{s - m_h^2} + \frac{1}{t - m_h^2} + \frac{1}{u - m_h^2} \right) - \lambda_{hhhh}, \quad (2.26)$$

with s, t, u denoting the standard Mandelstam variables defined in the center of mass frame. In particular, we also have $t = -(s - 4m_h^2) \sin^2 \frac{\theta}{2}$ and $u = -(s - 4m_h^2) \cos^2 \frac{\theta}{2}$, where \sqrt{s} is the center of mass energy and θ is the azimuthal angle with respect to the colliding axis.

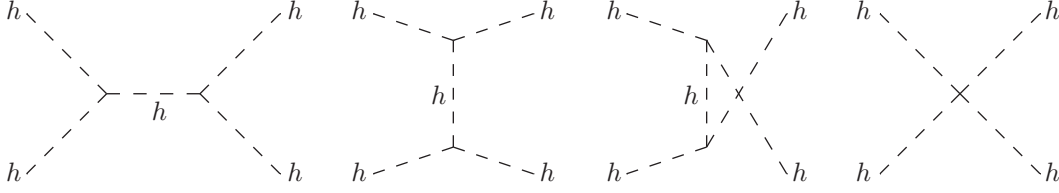


Figure 2. $hh \rightarrow hh$ scattering amplitudes: $s + t + u$ channels + 4-vertex (4vrtx) contributions.

The $J = 0$ partial wave is found to be

$$a_{hh \rightarrow hh}^0 = -\frac{1}{2} \frac{\sqrt{s(s-4m_h^2)}}{16\pi s} \left[\lambda_{hhh}^2 \left(\frac{1}{s-m_h^2} - 2 \frac{\log \frac{s-3m_h^2}{m_h^2}}{s-4m_h^2} \right) + \lambda_{hhhh} \right], \quad (2.27)$$

where we paid attention to keep the kinematical factors which makes the amplitude to vanish at threshold ($\sqrt{s} = 2m_h$) and we multiplied by an extra $1/2$ factor due to the presence of identical particles in the initial and final state (see e.g. [44] for a collection of relevant formulae). Following standard arguments [45, 46], perturbative unitarity bounds are obtained by requiring $|\text{Re } a_{hh \rightarrow hh}^0| < 1/2$.

The bound is displayed in Fig. 3 for the orthogonal cases in which either λ_{hhh} (upper plots) or λ_{hhhh} (lower plots) is modified with respect to the SM case. Note that the situation is qualitatively different for the two cases: being h^3 a relevant operator, the unitarity bound on λ_{hhh} is maximized at low energy, while in the case of h^4 the partial wave grows with energy reaching an asymptotic value at $\sqrt{s} \rightarrow \infty$.[§] In particular, from the right-side plots in Fig. 3 we read the following unitarity bounds

$$|\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}| \lesssim 6.5 \quad \text{and} \quad |\lambda_{hhhh}/\lambda_{hhhh}^{\text{SM}}| \lesssim 65. \quad (2.28)$$

Of course, one expects that new physics effects should modify at the same time both λ_{hhh} and λ_{hhhh} . However, since the h^3 and h^4 operators dominate the partial wave in two well-separated energy regimes they cannot cancel each other over the whole range of \sqrt{s} . Hence, since we require perturbativity at any value of \sqrt{s} , the bounds in Eq. (2.28) hold also in more general situations.

Let us inspect, for instance, the case where the modified SM potential arises from the operator $|H|^6$ as in Eq. (2.3). In such a case we have

$$\lambda_{hhh} = \lambda_{hhh}^{\text{SM}} + 6 c_6 v \simeq \lambda_{hhh}^{\text{SM}} (1 + 7.8 c_6), \quad (2.29)$$

$$\lambda_{hhhh} = \lambda_{hhhh}^{\text{SM}} + 36 c_6 \simeq \lambda_{hhhh}^{\text{SM}} (1 + 47 c_6). \quad (2.30)$$

The perturbativity bound coming from the h^3 (h^4) vertex in Eq. (2.28) translates into $|c_6| \lesssim 0.71$ (1.4).

[§]Note that this behaviour is different from the more standard case of effective operators, whose scattering amplitudes grow indefinitely with the energy.

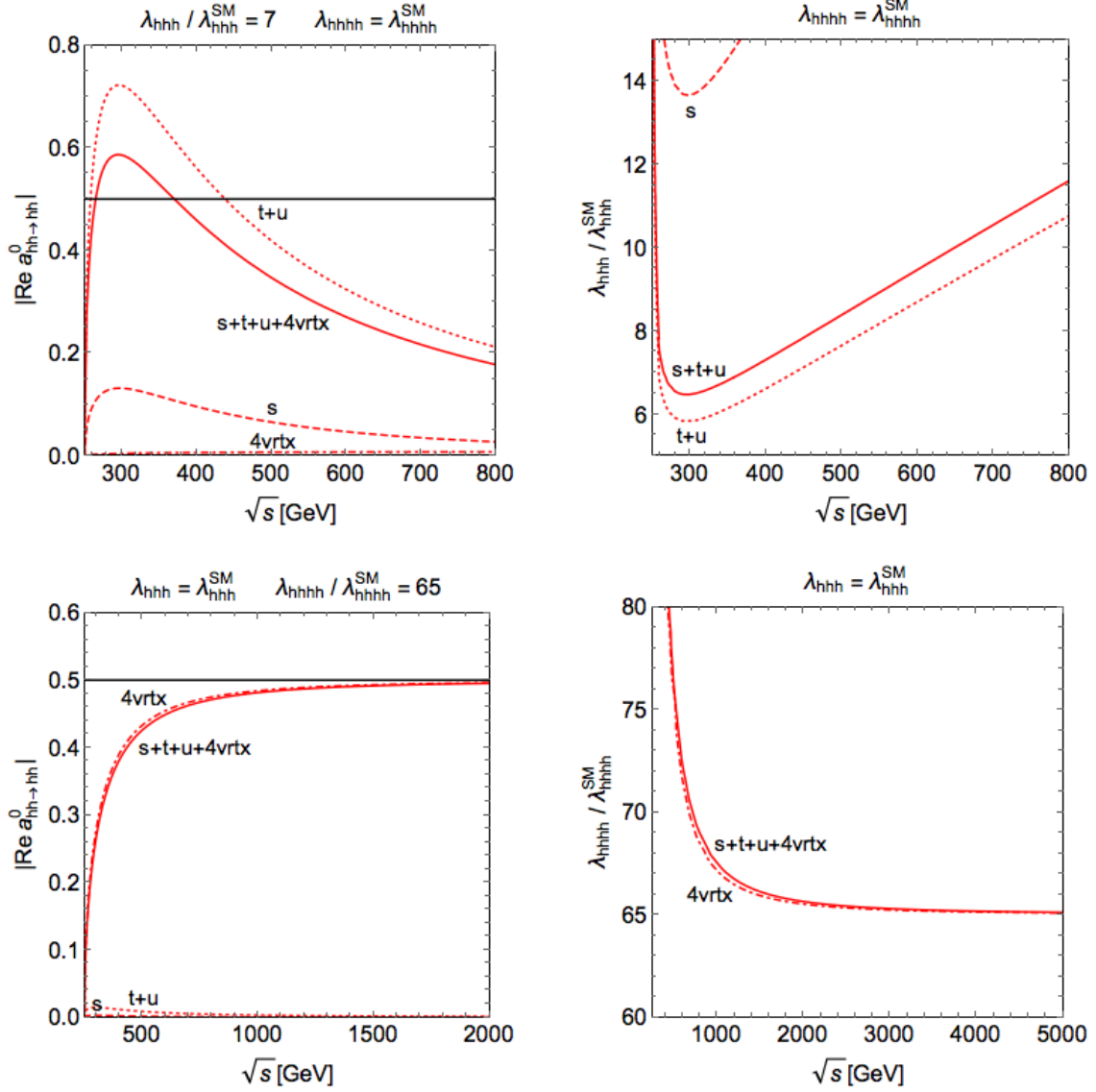


Figure 3. *Up/Left:* Kinematical dependence of $|\text{Re } a_{hh \rightarrow hh}^0|$ for the reference values $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} = 5$ and $\lambda_{hhhh} = \lambda_{hhhh}^{\text{SM}}$. *Up/Right:* Partial-wave unitarity bound $|\text{Re } a_{hh \rightarrow hh}^0| < 1/2$ on $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ as a function of \sqrt{s} and for $\lambda_{hhhh} = \lambda_{hhhh}^{\text{SM}}$. *Down/Left:* Kinematical dependence of $|\text{Re } a_{hh \rightarrow hh}^0|$ for the reference values $\lambda_{hhhh}/\lambda_{hhhh}^{\text{SM}} = 65$ and $\lambda_{hhh} = \lambda_{hhh}^{\text{SM}}$. *Down/Right:* Partial-wave unitarity bound $|\text{Re } a_{hh \rightarrow hh}^0| < 1/2$ on $\lambda_{hhhh}/\lambda_{hhhh}^{\text{SM}}$ as a function of \sqrt{s} and for $\lambda_{hhh} = \lambda_{hhh}^{\text{SM}}$. Dashed, dotted, dot-dashed and full curves denote respectively the s , $t+u$, 4vrtx and $s+t+u+4\text{vrtx}$ contribution to the partial wave. Note that s and 4vrtx have the opposite sign of $t+u$ (cf. Eq. (2.27)).

2.3.2 Loop-corrected vertices

Another perturbativity criterium that can be employed consists in the requirement that the loop-corrected trilinear scalar vertex is smaller (in absolute value) than λ_{hhh} . If that were

not the case, we clearly could not reliably use perturbation theory whenever λ_{hhh} entered some physical process. A similar criterium was employed for trilinear scalar interactions in Ref. [44], by setting to zero the external momenta of the 3-point function. By following the same argument, we obtain

$$\Delta\lambda_{hhh}(p_i \rightarrow 0) = \frac{1}{32\pi^2} \lambda_{hhh}^3 \frac{1}{m_h^2}. \quad (2.31)$$

By requiring that $|\Delta\lambda_{hhh}/\lambda_{hhh}| < 1$, this leads to the following perturbativity bound on the trilinear Higgs self-coupling

$$|\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}| \lesssim 12. \quad (2.32)$$

A stronger perturbativity bound can be obtained by looking at the full kinematical dependence of the trilinear vertex at the one-loop order. Considering the finite contribution due to λ_{hhh} we obtain

$$\Delta\lambda_{hhh}(\sqrt{s}, m_h) = -\frac{1}{16\pi^2} \lambda_{hhh}^3 C_0(m_h^2, m_h^2, s; m_h, m_h, m_h), \quad (2.33)$$

where C_0 is a scalar Passarino-Veltman function (defined according to the conventions of Ref. [47]) and \sqrt{s} denotes the off-shell momentum of a Higgs boson line. Note that we neglected divergent contributions due to quadrilinear scalar vertices and gauge couplings, which are reabsorbed by renormalization. Under the assumption that only the trilinear Higgs self-coupling is modified with respect to the SM, this is a valid approximation.

The perturbativity bound, denoted by λ_{hhh}^* , is shown in Fig. 4 as a function of \sqrt{s} . Note that above threshold, $\sqrt{s} > 2m_h$, C_0 develops an imaginary part and hence we have separately considered both the real and imaginary contribution to the bound. Since one should require that perturbativity must hold for any value of \sqrt{s} , the bound is maximized close to threshold and reads

$$|\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}| \lesssim 6, \quad (2.34)$$

which is consistent with the conceptually different bound obtained in Eq. (2.28).

A similar argument can be used to set a perturbativity bound on λ_{hhhh} by looking at its beta function (see e.g. [48]). By requiring $|\beta_{\lambda_{hhhh}}/\lambda_{hhhh}| < 1$, we get $|\lambda_{hhhh}| < \frac{16\pi^2}{3} \simeq 53$. Normalizing the latter with respect to the SM value implies

$$|\lambda_{hhhh}/\lambda_{hhhh}^{\text{SM}}| \lesssim 68, \quad (2.35)$$

which again is consistent with Eq. (2.28).

Given the impossibility of setting genuine model-independent bounds on λ_{hhh} beyond perturbativity, we focus in the next section on UV complete scenarios when investigating the question of the maximal value of the triple Higgs coupling. We focus for simplicity on weakly coupled models, as they retain a higher degree of predictivity and we have full control of the theory.

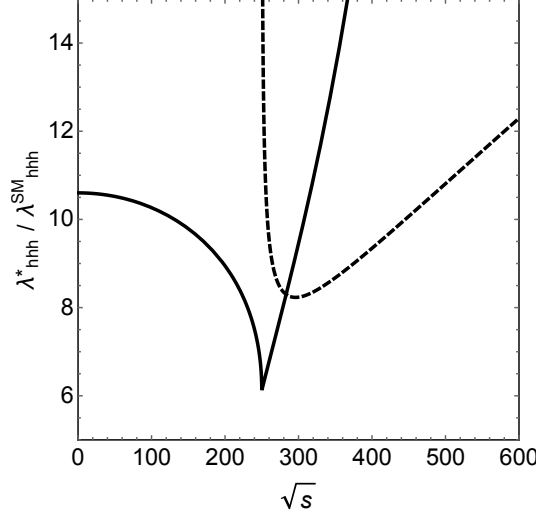


Figure 4. Perturbativity bound $\lambda_{hhh} < \lambda_{hhh}^*$ as a function of \sqrt{s} . Full and dashed curves denote respectively the real ($|\text{Re}(\Delta\lambda_{hhh})/\lambda_{hhh}| < 1$) and imaginary ($|\text{Im}(\Delta\lambda_{hhh})/\lambda_{hhh}| < 1$) contributions associated with the vertex correction in Eq. (2.33).

3 UV complete models

If the new degrees of freedom are very light, they can affect the Higgs-pair production process in different ways (like e.g. resonant production [49–56] or by scalar or fermionic contributions to the gluon fusion loop [57–59]) and the dominant effect does not need to be associated with the λ_{hhh} coupling deviation. Hence, we focus on the case where the new physics is above the EW scale, but not yet in the EFT regime where the effects are expected to decouple rapidly. The latter is nonetheless useful in order to classify the representations which are potentially more prone to induce a large effect: at tree level there are basically three class of diagrams (cf. Fig. 5) which can generate $|H|^6$ by integrating out a heavy new scalar degree of freedom.[¶] Note that we concentrate on trilinear Higgs self-couplings modifications generated by $|H|^6$, since they uniquely modify the Higgs self-couplings. Also the operator $\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H)$ gives a contribution to the shift in the trilinear Higgs self-coupling, but it modifies all other Higgs couplings as well.

In fact, the connecting motive between the diagrams in Fig. 5 turns out to be a tadpole operator of the type $\mathcal{O}_\Phi = \Phi f(H)$, where $f(H)$ is a string of Higgs fields (or their charged conjugates). The full list of scalar extensions that couple linearly to H can be found in Table 1 (see also Refs. [62–64]), where hyper-chargeless multiplets are understood to be real.

[¶]Note that it is also possible to exchange a massive vector at tree level, e.g. in presence of the trilinear coupling $g_V H^\dagger D_\mu H V^\mu$, where V^μ has gauge quantum numbers (1, 1, 0) or (1, 3, 0) (see e.g. [60, 61]). After integrating V^μ out and applying the equations of motion one obtains an $|H|^6$ operator with Wilson coefficient proportional to $\lambda g_V^2/M_V^2$. On the other hand, massive vectors (either in their gauge extended or strongly coupled version) require a UV completion, thus going beyond our simplifying assumption of a one-particle extension of the SM.

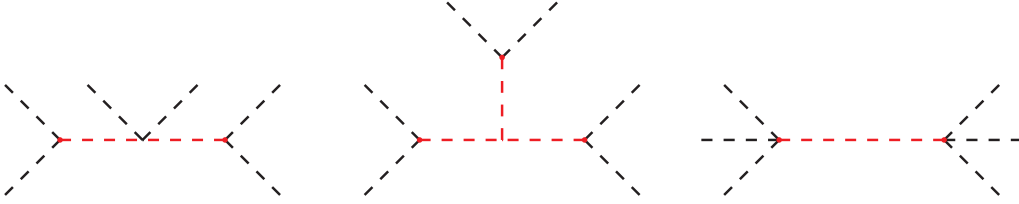


Figure 5. Tree-level generation of the $|H|^6$ operator (external lines, black) obtained by integrating out new scalar degrees of freedom (internal propagators, red).

For simplicity, we will focus on one-particle extensions of the SM in order to identify their features in a clear way.

Φ	\mathcal{O}_Φ
$(1, 1, 0)$	$\Phi H H^\dagger$
$(1, 2, \frac{1}{2})$	$\Phi H H^\dagger H^\dagger$
$(1, 3, 0)$	$\Phi H H^\dagger$
$(1, 3, 1)$	$\Phi H^\dagger H^\dagger$
$(1, 4, \frac{1}{2})$	$\Phi H H^\dagger H^\dagger$
$(1, 4, \frac{3}{2})$	$\Phi H^\dagger H^\dagger H^\dagger$

Table 1. List of new scalars Φ inducing a tree-level modification of the triple-Higgs coupling via the tadpole operator \mathcal{O}_Φ .

Another useful way to understand the origin of the trilinear Higgs self-coupling modification, which does not rely on the EFT language is the following: the tadpole operator will unavoidably generate a vev for Φ , and hence the neutral components $h^0 \subset H$ and $\phi^0 \subset \Phi$ will mix via the tadpole operator itself. After projecting the two neutral components on the Higgs boson mass eigenstate, namely $h^0 \rightarrow h \cos \theta$ and $\phi^0 \rightarrow h \sin \theta$, we have the following contribution to the triple-Higgs vertex

$$\Delta \lambda_{hhh} = \mu_\Phi \sin \theta \cos^2 \theta \quad \text{or} \quad \lambda_\Phi v \sin \theta \cos^3 \theta, \quad (3.1)$$

depending whether the tadpole operator is $d = 3$ (μ_Φ coupling) or $d = 4$ (λ_Φ coupling). Since there is a single suppression from the mixing angle, bounded at the level of $\theta \lesssim 0.3$ from Higgs coupling measurements, the tadpole interaction is expected to yield the largest contribution, while other mixing operators in the scalar potential entail extra suppressions from $\sin \theta$. We

can also naively estimate the contribution in the following way: assuming that $\mu_\Phi/v \lesssim 4\pi$ and $\lambda_\Phi \lesssim 4\pi$ by perturbativity we get

$$\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}} \lesssim 4\pi \sin\theta \cos^2\theta \frac{v^2}{3m_h^2} \sim 4. \quad (3.2)$$

To make this estimate more precise, we will look in detail at two paradigmatic examples among those in Table 1: one model which exhibits a tree-level custodial symmetry (singlet case, Sect. 3.1) and one which does not (triplet case, Sect. 3.2).

A notable feature of tadpole interactions is that, being “odd” in Φ and H , they are potentially bounded by vacuum stability considerations. Remarkably, we find that vacuum stability is never a crucial discriminant for bounding the largest value of λ_{hhh} , because whenever the tadpole coupling is large the instability can be tamed by large (within the perturbativity domain) quartic couplings. For this reason we find it relevant to discuss in Sect. 3.3 a class of loop-induced trilinear Higgs self-couplings that arise due to vector-like fermions, where one can establish a direct connection between λ_{hhh} and the vacuum instability.

3.1 Tree-level custodially symmetric cases

Among the cases in Table 1, the singlet and the doublet do not violate custodial symmetry at tree level and hence have the chance to yield the largest contribution to λ_{hhh} . We will discuss in detail the singlet, while we only comment on the case of the doublet towards the end of the subsection. The scalar potential reads

$$V(H, \Phi) = \mu_1^2 |H|^2 + \lambda_1 |H|^4 + \frac{1}{2} \mu_2^2 \Phi^2 + \mu_4 |H|^2 \Phi + \frac{1}{2} \lambda_3 |H|^2 \Phi^2 + \frac{1}{3} \mu_3 \Phi^3 + \frac{1}{4} \lambda_2 \Phi^4, \quad (3.3)$$

where we have omitted a tadpole term for the singlet field, as it can be reabsorbed in the singlet vev by a field redefinition.

In fact, the μ_4 coupling unavoidably induces a vev for Φ and also leads to a mixing between H and Φ . In Appendix A.1 we give the tadpole equations and we define the mixing angle θ between the singlet and doublet fields. Some of the parameters of the potential can be expressed in terms of the physical masses and vevs and their mixing angle. We chose as input parameters

$$v_H = 246.2 \text{ GeV}, \quad v_S, \quad m_1 = 125 \text{ GeV}, \quad m_2, \quad \theta, \quad \lambda_2, \quad \lambda_3. \quad (3.4)$$

Their relations to the other parameters of the potential can be found in Appendix A.1. Note that the scenario in which the SM-like Higgs boson is heavier than the singlet-like scalar is phenomenologically viable as well, but we will restrict ourselves to the case $m_1 \ll m_2$. The reason being that we want to discuss deviations to the Higgs pair production process that are mainly stemming from the trilinear Higgs self-coupling, where the contribution from an exchange of the singlet-like Higgs boson in the triangle diagrams is suppressed. For discussion on resonant Higgs pair production in the singlet model we refer to Refs. [49–56].

The trilinear Higgs self-coupling is given by

$$\begin{aligned}
\lambda_{hhh} &= 6\lambda_1 v_H \cos^3 \theta - (3\mu_4 + 3\lambda_3 v_S) \cos^2 \theta \sin \theta + 3\lambda_3 v_H \cos \theta \sin^2 \theta - \sin^3 \theta (2\mu_3 + 6v_S \lambda_2) \\
&= \lambda_{hhh}^{\text{SM}} \cos \theta \left[1 + \sin^2 \theta \left(\frac{\lambda_3 v_H^2}{m_1^2} - 1 \right) + \sin^4 \theta \frac{v_H^2}{3v_S^2} \left(1 - \frac{m_2^2}{m_1^2} \right) \right. \\
&\quad \left. - \frac{v_H}{3v_S} \frac{\sin^3 \theta}{\cos \theta} \left(2 \sin^2 \theta + 2 \cos^2 \theta \frac{m_2^2}{m_1^2} - \frac{\lambda_3 v_H^2}{m_1^2} + \frac{2v_S^2 \lambda_2}{m_1^2} \right) \right], \tag{3.5}
\end{aligned}$$

where in the last step we expressed λ_{hhh} in terms of the input parameters in Eq. (3.4).

In order to tie to the discussion at the beginning of Sect. 3 on the importance of tadpole operators for enhancing the trilinear Higgs self-coupling, let us compare the expression in Eq. (3.5) with the one obtained in the \mathbb{Z}_2 -symmetric limit with $\mu_{3,4} \rightarrow 0$, which yields

$$\lambda_{hhh}^{\mathbb{Z}_2\text{-symmetric}} = \lambda_{hhh}^{\text{SM}} \left(\cos^3 \theta - \sin^3 \theta \frac{v_H}{v_S} \right). \tag{3.6}$$

It is thus evident that the shift in the trilinear Higgs self-coupling can be much larger for the general singlet potential with tadpole terms.^{||} In the last step of Eq. (3.5) we see indeed that potentially large contributions can arise from sizable values of λ_3 , which is bounded by perturbativity.

In the following we will discuss which values the trilinear Higgs self-coupling can take, by accounting for several constraints.

3.1.1 Indirect bounds

The model parameters can be restricted by EW precision tests, Higgs coupling measurements, perturbativity arguments and vacuum stability. These will then indirectly constrain the trilinear Higgs self-coupling in the model.

EW precision tests:

In Ref. [67] it was pointed out that the measurement of the W boson mass constrains the scalar singlet model more strongly than a fit on the S , T , U parameters. Even though the study in Ref. [67] concerns a \mathbb{Z}_2 symmetric potential, we can use the bounds here, since at the one-loop order the additional parameters in the scalar potential do not play any role for the gauge boson vacuum polarizations. For $m_2 > 800$ GeV, Ref. [67] finds the bound $|\sin \theta| < 0.2$.

Higgs coupling measurements:

The Higgs production and decay rates are modified with respect to the SM by a universal factor

$$\sigma(pp \rightarrow h + X) = \cos^2 \theta \sigma_{\text{SM}}(pp \rightarrow h + X), \tag{3.7}$$

$$\Gamma(h \rightarrow XX) = \cos^2 \theta \Gamma_{\text{SM}}(h \rightarrow XX). \tag{3.8}$$

^{||}For comparison, in the \mathbb{Z}_2 -symmetric case one finds that the maximal deviations on the trilinear Higgs self-coupling are at the 10% level, in the case where the second Higgs boson cannot be directly detected at the LHC [65, 66].

The branching ratios of the SM-like Higgs boson are not modified compared to the SM, if it is the lighter one. In Ref. [68] a limit on $\sin^2 \theta < 0.12$ at 90% C.L. from Higgs signal measurements is given. This limit turns out to be stronger than the limits from direct searches of the heavier Higgs boson, as long as $m_2 > 450$ GeV [69], such that we will not need to take the latter into account for the parameter space we consider.

Perturbativity:

For large enough potential couplings unitarity is violated in tree-level scattering processes, thus signalling the breakdown of perturbation theory. Simple criteria can be derived from the $ii \rightarrow jj$ scattering, with i and j running over the (real) Higgs and singlet fields. By requiring $|\text{Re } a_0| < 1/2$ for the eigenvalues of the $J = 0$ partial-wave scattering matrix, we derive the following constraint in the high-energy limit

$$3(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + \lambda_3^2} < 16\pi. \quad (3.9)$$

The dimensionful parameters μ_3 and μ_4 can be restricted by unitarity arguments as well. However, being associated to super-renormalizable operators the bounds are maximized at low energies, where the possible presence of resonances actually requires a careful treatment of the pole singularities. Following the argument of Ref. [44], in order to define the perturbative domain of μ_3 and μ_4 we require instead that the one-loop corrected trilinear scalar couplings at zero external momenta remain smaller than the tree-level ones. In the $SU(2)$ limit we obtain

$$\frac{|\mu_4|}{\max(|\mu_1|, |\mu_2|)} < 4\pi, \quad \wedge \quad \left| \frac{\mu_3}{\mu_2} \right| < 4\pi. \quad (3.10)$$

The saturation of the bounds in Eqs. (3.9)–(3.10) correspond to an extreme situation, where we progressively enter a strongly-coupled regime for which the perturbative calculation does not make sense anymore. For this reason, we will also present the results in a safely weakly-coupled regime, roughly corresponding to the replacement $4\pi \rightarrow 1$ in Eqs. (3.9)–(3.10).

Vacuum stability:

The requirement that the scalar potential is bounded from below imposes the following conditions on the quartic scalar interactions

$$\lambda_1 > 0, \quad \wedge \quad \lambda_2 > 0, \quad \wedge \quad \lambda_3 > -2\sqrt{\lambda_2 \lambda_1}. \quad (3.11)$$

The study of the minima of the scalar potential exhibits a rich structure, with new local minima (e.g. in $h = 0$) that arise in some regions of the parameter space and which might eventually destabilize the EW vacuum. A detailed analysis of the vacuum structure at tree level can be found in Refs. [51, 70]. We check for vacuum stability by using **Vevacious** [71, 72], with a model file generated with **SARAH** [73–77].

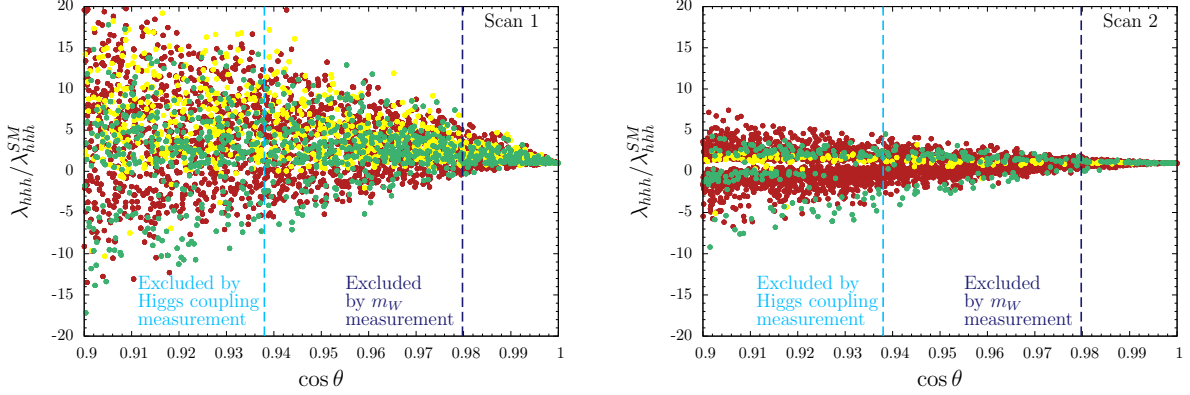


Figure 6. *Left:* The trilinear Higgs self coupling normalised to the SM reference value for scan 1 (strongly-coupled regime). The red/yellow/green points correspond respectively to unstable/metastable/stable configurations. The dashed vertical lines indicate the bounds on $\cos \theta$ from the respective experimental measurements. *Right:* Same as in left panel, but for scan 2 (weakly-coupled regime).

3.1.2 Results

In order to show results we perform a scan over the parameter space. We will perform two different scans. The universally scanned parameters in both the cases are

$$\begin{aligned} m_1 &= 125 \text{ GeV}, & 800 \text{ GeV} < m_2 < 2000 \text{ GeV}, \\ v_H &= 246.2 \text{ GeV}, & |v_S| < m_2, & 0.9 < \cos \theta < 1. \end{aligned} \quad (3.12)$$

In the first scan we use the maximally allowed values according to the perturbative argument

$$\text{Scan 1:} \quad 0 < \lambda_2 < \frac{8}{3}\pi, \quad |\lambda_3| < 16\pi, \quad (3.13)$$

and reject all points that do not fulfil Eq. (3.9), Eq. (3.10) and Eq. (3.11). In the second scan we restrict ourselves to weakly-coupled scenarios, meaning that

$$\text{Scan 2:} \quad 0 < \lambda_2 < 1/6, \quad |\lambda_3| < 1, \quad (3.14)$$

and $|\mu_4|/\max(|\mu_1|, |\mu_2|) < 1$ and $|\mu_3/\mu_2| < 1$.

In Fig. 6 the trilinear Higgs self coupling normalised to the SM coupling is shown. The color code of the points indicate whether they correspond to a stable, metastable or unstable vacuum configuration. By accounting for the bounds of the m_W boson measurement we find the following range for the allowed trilinear Higgs self-coupling:

$$\text{Scan 1:} \quad -1.5 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 8.7, \quad (3.15)$$

$$\text{Scan 2:} \quad -0.3 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 2.0. \quad (3.16)$$

In fact, the largest value of the trilinear Higgs self-coupling is crucially related to the perturbativity domain. The bounds on the trilinear Higgs self-coupling obtained from scan 1 should hence be treated with care, as they are very close to the perturbative regime and loop corrections can be expected to be large. This can be easily understood looking at the formulae in Eq. (3.5). By allowing for rather large values of e.g. λ_3 we can get much larger deviations. Note that we find here a larger value for $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ as in Sect. 2.3, since we require a weaker perturbativity criterium in (3.10) corresponding to the one in Eq. (2.32). On the other hand, as it can be inferred from Fig. 6, the requirement of a stable vacuum has only a very small impact on the bound of the trilinear Higgs self-coupling. The little impact of vacuum stability can be understood by the fact that the presence of many parameters in the scalar potential basically uncorrelates the stability conditions from the trilinear Higgs self-coupling.

At this point, we would like to comment on previous studies in the context of the scalar singlet. In Ref. [78], deviations for $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ up to -10 were found. Note however that much weaker limits on the mixing angle θ were employed, since the bound stemming from the m_W measurement was not used. In addition, weaker bounds from the Higgs coupling measurements were employed. In Ref. [79, 80] one-loop corrections to the trilinear Higgs self-coupling were computed. They can give large corrections (even up to 100%) from non-decoupling effects in the Higgs boson loops if $\lambda_3 v_H^2 \gg \mu_2^2$ [81]. This is not surprising, given the fact that one is saturating the perturbativity limit where loop effects are not under control.

We conclude with a few remarks on the other custodial symmetric case, namely the two-Higgs doublet model (2HDM). The question of the trilinear Higgs self-coupling was addressed in detail in the context of the \mathbb{Z}_2 symmetric case [82, 83], where it was shown that the expected deviations are well below those allowed in the general singlet model. On the other hand, a full study in the context of the general 2HDM (including the $\Phi H H^\dagger H^\dagger$ tadpole operator) is still missing to our knowledge (see however [84] for a qualitative study). In such a case we expect potentially large deviations. We leave this study for future investigations.

3.2 Tree-level custodially violating cases

We shall discuss the cases corresponding to the last four rows in Table 1 altogether, since they have in common the fact that the tadpole term $\Phi f(H)$ contributing to a potentially sizable triple Higgs self-coupling generates a custodial-breaking vev for Φ , which is strongly bounded by EW precision tests.

Let us exemplify the analysis for the case of a real EW triplet with zero hypercharge, $\Phi \sim (1, 3, 0)$. The scalar potential reads (see e.g. [85])

$$V(H, \Phi) = \mu_1^2 |H|^2 + \frac{1}{2} \mu_2^2 |\Phi|^2 + \lambda_1 |H|^4 + \frac{1}{4} \lambda_2 |\Phi|^4 + \frac{1}{2} \lambda_3 |H|^2 |\Phi|^2 + \mu_4 H^\dagger \sigma^\alpha H \Phi^\alpha, \quad (3.17)$$

where, without loss of generality, we can take $\mu_4 > 0$ by reabsorbing the sign in the definition of Φ . The minimization of the potential and the calculation of the scalar spectrum is deferred to Appendix A.2. In particular, we can choose the following independent observables as

parameter inputs for the model

$$v_H = \sqrt{v^2 - 4v_T^2}, \quad v_T < 3.5 \text{ GeV}, \quad m_1 = 125 \text{ GeV}, \quad m_2, \quad m_{h^\pm}, \quad \theta, \quad (3.18)$$

where $v = 246.2 \text{ GeV}$. The trilinear Higgs self-coupling is given by

$$\begin{aligned} \lambda_{hhh} &= 6\lambda_1 v_H \cos^3 \theta + 3(\mu_4 - \lambda_3 v_T) \cos^2 \theta \sin \theta + 3\lambda_3 v_H \cos \theta \sin^2 \theta - 6\lambda_2 v_T \sin^3 \theta \quad (3.19) \\ &= \frac{3m_1^2}{v_H} \cos \theta \left[1 + \left(\frac{2m_{h^\pm}^2 v_H^2}{(v_H^2 + 4v_T^2)m_1^2} - 1 \right) \sin^2 \theta + \left(\frac{m_{h^\pm}^2 v_H^2}{(v_H^2 + 4v_T^2)m_1^2} - 1 \right) \frac{v_H \sin^3 \theta}{v_T \cos \theta} \right], \end{aligned}$$

where in the last step we expressed λ_{hhh} in terms of the parameters in Eq. (3.18).

3.2.1 Indirect bounds

As in the singlet case, we are going to consider in turn EW precision tests, Higgs coupling measurements, perturbativity arguments and vacuum stability in order to constrain the trilinear Higgs self-coupling in the model.

EW precision tests:

The main bound comes from the tree-level modification of the ρ parameter. In the SM the custodial symmetry of the Higgs potential ensures the tree-level relation $\rho \equiv m_W^2/m_Z^2 \cos^2 \theta_W = 1$. Extra sources of custodial symmetry breaking which cannot be accounted within the SM are described by the $\rho_0 \equiv \rho/\rho_{\text{SM}}$ parameter. Provided that the new physics which yields $\rho_0 \neq 1$ does not significantly affect the SM radiative corrections,** a global fit to EW observables yields $\rho_0^{(\text{fit})} = 1.00037 \pm 0.00023$ [88]. In the triplet model one has

$$\rho_0^{\text{tree}} = 1 + 4 \frac{v_T^2}{v_H^2}, \quad (3.20)$$

and using the 2σ -level bound from $\rho_0^{(\text{fit})}$ we obtain $v_T < 3.5 \text{ GeV}$.

Higgs coupling measurements:

In case of a triplet, the Higgs couplings are modified by $\cos \theta$, while the gauge-Higgs boson couplings get a contribution from the triplet admixture proportional to $\sin \theta$. The mixing angle between the doublet and triplet scalar fields is necessarily rather small since $\theta \rightarrow 0$ for $v_T/v_H \rightarrow 0$. This means that the tree-level Higgs couplings to fermions and gauge bosons are basically unmodified. The charged Higgs boson contributes to the loop-induced $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$ decay. Its contribution is however negligible for $m_{h^\pm} \gtrsim 300 \text{ GeV}$ [63]. Perturbativity requirements and EW precision tests lead to rather small mass splittings of $\mathcal{O}(\text{few GeV})$ between the neutral and charged components of the triplet. Since we are interested in a non-resonant region of phase space for the Higgs pair production process, we consider

**This does not need to be the case in models with $\rho \neq 1$ at tree level, where four input parameters (instead of three) are required for the EW renormalization [85–87]. An investigation of this issue is however beyond the scope of this paper.

scenarios with significantly larger charged Higgs boson masses m_{h^\pm} and m_2 . Furthermore, we check for exclusion limits of additional Higgs bosons by means of the code **HiggsBounds** [89–91]. It turns however out that for our parameter scan, no points are excluded.

Perturbativity:

The adimensional couplings in the potential of Eq. (3.17) are bounded by perturbative unitarity. Looking at correlated matrix of $2 \rightarrow 2$ scattering processes one finds [92]

$$\lambda_1 < 4\pi, \quad \lambda_2 < 4\pi, \quad \lambda_3 < 8\pi, \quad 6\lambda_1 + 5\lambda_2 \pm \sqrt{(6\lambda_1 - 5\lambda_2)^2 + 12\lambda_3^2} < 16\pi. \quad (3.21)$$

For the dimensionful parameter μ_4 we estimate the finite loop corrections to the μ_4 vertex at zero external momenta and require it to be smaller than the tree-level value. In the $SU(2)_L$ limit we obtain

$$\frac{|\mu_4|}{\max(|\mu_1|, |\mu_2|)} < 4\pi. \quad (3.22)$$

Vacuum stability:

By requiring that the potential is bounded from below, we obtain the conditions

$$\lambda_1 > 0, \quad \wedge \quad \lambda_2 > 0, \quad \wedge \quad \lambda_3 > -2\sqrt{\lambda_1\lambda_2}. \quad (3.23)$$

Also the massive coupling μ_4 can destabilize the potential, if too large. We check for vacuum stability using **Vevacious** [71, 72], with a model file generated with **SARAH** [73–77].

In principle, there can also be charge breaking (CB) minima. For a CB stationary point we find the necessary condition

$$v_{CB}^{\eta_+} \left(\frac{\lambda_3}{2} v_{H,CB}^2 + \mu_2^2 + \lambda_2 v_{T,CB}^2 + 2\lambda_2 |v_{CB}^{\eta_+}|^2 \right) = 0, \quad (3.24)$$

where the subscript “CB” refers to the vevs in the CB minimum and $\langle \eta_+ \eta_- \rangle = |v_{CB}^{\eta_+}|^2$. In addition, from the other stationary equations we find that $v_{H,CB} = 0$ for $v_{CB}^{\eta_+} \neq 0$ (if $\mu_4 \neq 0$). Hence, Eq. (3.24) implies that non-zero CB stationary points can exist only if

$$\frac{1}{2\lambda_2} (\mu_2^2 + \lambda_2 v_{T,CB}^2) < 0. \quad (3.25)$$

Since $\lambda_2 > 0$ from the boundedness of the potential, there are no CB stationary points as long as $\mu_2^2 > 0$. We checked explicitly that for all our parameter points $\mu_2^2 > 0$. This can be explained as follows. For $v_T/v_H \ll 1$, we can approximate

$$\mu_2^2 \simeq -\frac{\sin 2\theta (m_2^2 - m_1^2) v_H}{4v_T} \quad \text{and} \quad \tan 2\theta \simeq \frac{4v_T}{v_H \mu_4} (\lambda_3 v_T - \mu_4). \quad (3.26)$$

Since we work in the basis where $\mu_4 > 0$, the requirement that $m_{h^\pm}^2 > 0$ implies $v_T > 0$ (cf. Eq. (A.21)). In our scan we use $m_{h^\pm} > 800$ GeV. From that we can compute a lower bound on μ_4/v_T by using Eq. (A.31). Due to the perturbativity bound on λ_3 , i.e. $\lambda_3 < 8\pi/\sqrt{3}$, from Eq. (3.21) one then finds that $(\lambda_3 - \mu_4/v_T) < 0$. Hence, for our scan $\mu_2^2 > 0$ and we do not need to care for CB minima.

3.2.2 Results

As for the singlet, we perform a scan over the parameter space. The scan parameters are

$$\begin{aligned} m_1 = 125 \text{ GeV}, \quad 800 \text{ GeV} < m_{h^\pm} < 4000 \text{ GeV}, \quad v = 246.2 \text{ GeV}, \\ 0 < v_T < 3.55 \text{ GeV}, \quad 0.95 < \cos \theta < 1, \quad 0 < \lambda_2 < 4\pi. \end{aligned} \quad (3.27)$$

It turns out that it is better to scan over λ_2 rather than m_2 since the mass difference between m_2 and m_{h^\pm} is small due to the perturbativity requirement on λ_2 (cf. Eq. (A.33)). In Fig. 7

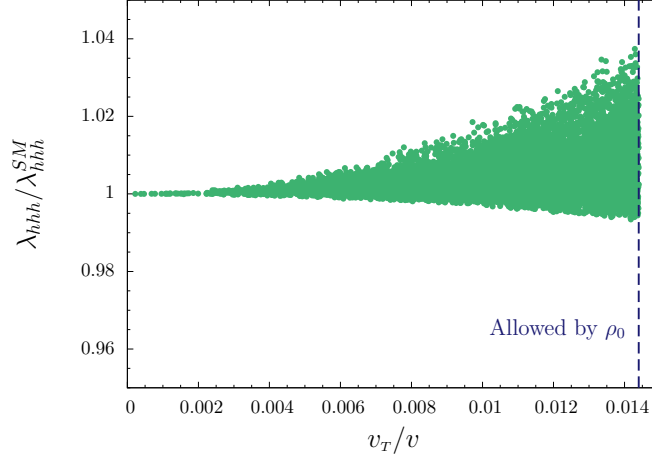


Figure 7. The modification of the trilinear Higgs self-coupling with respect to the SM as a function of v_T/v . For all points the minimum (v_H, v_T) is the global one.

we show the results of our parameter scan. The trilinear Higgs self-coupling can only be modified by a few percent in the triplet model. This is a consequence of the small values for v_T/v_H allowed by EW precision data.

As it can be inferred from the plot all points are stable at tree level. That can be understood as follows. In the neutral direction of H the potential has stationary points in $\langle H \rangle = 0$ and $\langle H \rangle = v_H/\sqrt{2}$. For $\langle H \rangle = v_H/\sqrt{2}$ the derivative of the potential with respect to the neutral component η^0 of Φ reads

$$\frac{\partial V}{\partial \eta_0} = \lambda_2 \eta_0^3 + \left(\mu_2^2 + \frac{\lambda_3}{2} v_H^2 \right) \eta_0 - \frac{\mu_4}{2} v_H^2 = 0. \quad (3.28)$$

The discriminant then reads

$$\Delta = -4\lambda_2 \left(\mu_2^2 + \frac{\lambda_3}{2} v_H^2 \right)^2 - \frac{27}{4} \lambda_2^2 \mu_4^2 v_H^4, \quad (3.29)$$

and $\Delta < 0$ for all parameter sets due to the boundedness from below condition on λ_2 from eq. (3.23), hence there are no further stationary points with $\langle H \rangle = v_H/\sqrt{2}$ in H direction.

Note that for the singlet in Sect. 3.1 due to the S^3 term in the potential, the discriminant can be either smaller or larger than 0.

Two further stationary points are possible, namely $(\langle H \rangle = 0, \langle \Phi \rangle = 0)$ and $(\langle H \rangle = 0, |\langle \Phi \rangle|^2 = -\mu_2^2/\lambda_2)$. Since we always find $\mu_2^2 > 0$ in our scan the latter is not relevant here and $(\langle H \rangle = 0, \langle \Phi \rangle = 0)$ must be a maximum by construction.

It is instructive to compare the previous results with the EFT limit where the triplet mass parameter is $\mu_2 \gg v$. By integrating out the triplet in the $SU(2)_L$ limit via the equations of motion

$$\Phi^\alpha \simeq -\frac{\mu_4}{\mu_2^2 + \lambda_3 |H|^2} H^\dagger \sigma^\alpha H \quad (3.30)$$

the potential in the EFT reads

$$V_{\text{EFT}}(H) \simeq -\frac{1}{2} \frac{\mu_4^2}{\mu_2^2 + \lambda_3 |H|^2} |H|^4 = -\frac{\mu_4^2}{2\mu_2^2} |H|^4 + \frac{\mu_4^2 \lambda_3}{2\mu_2^4} |H|^6 + \dots, \quad (3.31)$$

where the expansion in the last term holds for Higgs fluctuations around the EW vev. The first term in Eq. (3.31) simply redefines the Higgs quartic coupling in the SM EFT, while the second one yields

$$c_6 = \frac{\mu_4^2 v_H^2 \lambda_3}{2\mu_2^4}. \quad (3.32)$$

Always working in the $\mu_2 \gg v$ limit, we can approximate the triplet vev as (cf. Eq. (A.19))

$$v_T \simeq \frac{\mu_4 v_H^2}{2\mu_2^2}. \quad (3.33)$$

Hence, it is possible to recast the modified triple Higgs coupling as

$$\frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}} = 1 + \frac{2c_6 v_H^2}{m_h^2} = 1 + \frac{4v_T^2 \lambda_3}{m_h^2}, \quad (3.34)$$

where in the last step we have replaced c_6 in terms of v_T (cf. Eqs. (3.32)–(3.33)). By plugging $v_T \lesssim 3.5$ GeV and $\lambda_3 \in [-2\sqrt{0.1 \times 4\pi}, 8\pi/\sqrt{3}]$ from perturbativity and vacuum stability (also, $\lambda_1 \sim 0.1$ in order to reproduce the Higgs mass), we get $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} \in [0.993, 1.046]$, which fairly describes the range of deviations in Fig. 7.

A final comment on the other custodially violating cases is in order here. By denoting the vev of the complex multiplet as $\langle \Phi \rangle = v_\Phi/\sqrt{2}$, the 2σ -level bound from $\rho_0^{(\text{fit})}$ implies $v_\Phi \lesssim 1.7, 2.9, 1.0$ GeV, respectively for the $\Phi = (1, 3, 1), (1, 4, \frac{1}{2}), (1, 4, \frac{3}{2})$. We hence expect suppressed contributions for the trilinear Higgs self-coupling, similarly to the triplet case.

3.3 Loop-induced trilinear Higgs self-coupling vs. vacuum stability

Loop modifications of the trilinear Higgs self-coupling are naturally expected to be smaller than tree-level ones. Nevertheless, we consider here the case where the new particles circulating in the loops are vector-like fermions, since we regain a clean correlation between the triple Higgs coupling and vacuum instability. This can be easily understood by looking at the loop

of fermions contributing to the beta function of the Higgs self-coupling, which is basically the same diagram responsible for the radiative generation of the trilinear Higgs self-coupling in the broken phase after taking one Higgs to its vev (cf. Fig. 8).

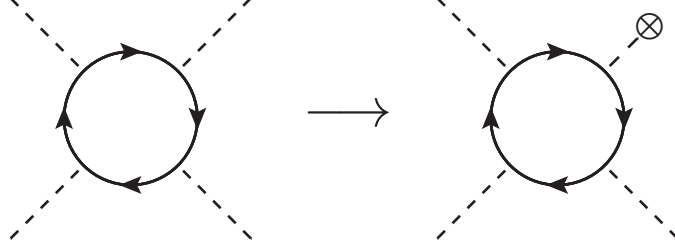


Figure 8. Schematic view of the connection between the beta-function of λ and the loop-induced trilinear Higgs self-coupling via new fermions.

There are basically two qualitatively different possibilities: *i*) non-SM-singlet fermions coupling to the Higgs and a SM fermion and *ii*) SM-singlet fermions coupling to the Higgs and a lepton doublet. The former cases are bounded by other Higgs coupling measurements, which typically imply a very suppressed contribution to the trilinear Higgs self-coupling. The latter is more interesting, and correspond to the case of a right handed neutrino, which is largely unconstrained by other Higgs coupling measurements. A recent analysis was performed in Refs. [93, 94] in the context of a simplified 3 + 1 Dirac neutrino model [93] and for the inverse seesaw model [94], finding deviations of the trilinear Higgs self-coupling with respect to the SM value up to 30%.

We want to show here the impact of vacuum stability in such a class of scenarios. Let us consider, for definiteness, the case of the inverse seesaw (similar conclusions apply to other neutrino mass models as well). We add to the SM field content three right-handed neutrinos and three gauge singlets X with opposite lepton number, via the Lagrangian term

$$\mathcal{L}_{\text{ISS}} = -Y_\nu \bar{L} \tilde{\Phi} \nu_R - M_R \bar{\nu}^c X - \frac{1}{2} \mu_X \bar{X}^c X + \text{h.c.}, \quad (3.35)$$

where we suppressed family indices. We refer to Ref. [94] for the relevant notation and conventions. Taking, in particular, a diagonal Yukawa structure $Y_\nu = |y_\nu| I_3$ and a common mass scale for the three heavy neutrinos, $M_R = 10$ TeV, one can assess the impact of the heavy neutrino states on the running of the Higgs self-coupling and hence on the stability of the Higgs effective potential $V_{\text{eff}}(h) \approx 1/4 \lambda_{\text{eff}}(h) h^4$, where $\lambda_{\text{eff}}(h)$ is approximated with the $\overline{\text{MS}}$ running coupling $\lambda(\mu = h)$. We use the two-loop beta functions for the SM couplings ($g_{1,2,3}, y_t, \lambda$) and take into account the corrections due to y_ν at the one-loop level (and consistently we neglect the matching contributions of y_ν to $\lambda(M_t)$). For simplicity, we also integrate in the heavy neutrinos at the common threshold $M_R = 10$ TeV, while a more careful treatment should take into account intermediate EFTs when integrating in single neutrino thresholds (see e.g. Ref. [95]). Hence, in the case of a hierarchical heavy neutrino spectrum, our estimate

of the largest energy scale until which the model can be consistently extrapolated should be conservatively rescaled starting from the heaviest threshold.

The results are displayed in Fig. 9 where we plot the value of λ_{eff} as a function of the renormalization scale μ . The instability bound (red area) is computed by considering the probability of decay against quantum tunnelling in the modified Higgs potential integrated over the past light-cone (see e.g. [96, 97])

$$\mathcal{P}_{\text{EW}} \simeq \left(\frac{\mu}{H_0} \right)^4 e^{-\frac{8\pi^2}{3|\lambda_{\text{eff}}(\mu)|}}, \quad (3.36)$$

where $H_0 \simeq 10^{-42}$ GeV is the present Hubble constant. In particular, requiring $\mathcal{P}_{\text{EW}} \simeq 1$ corresponds to

$$|\lambda_{\text{eff}}(\mu)| \simeq \frac{0.064}{1 + 0.022 \log_{10} \left(\frac{\mu}{1 \text{ TeV}} \right)}, \quad (3.37)$$

which sets the instability bound for $\lambda_{\text{eff}} < 0$.

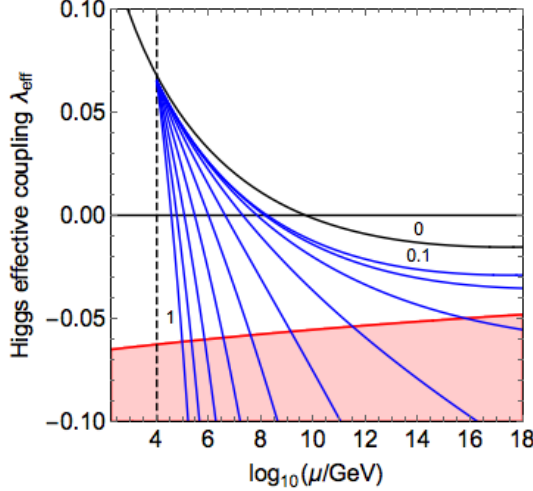


Figure 9. Running of λ_{eff} in the presence of a common heavy neutrino threshold $M_R = 10$ TeV. Labels denote the value of $y_\nu \in [0.1, 1]$ with steps of 0.1 (blue curves), while $y_\nu = 0$ corresponds to the SM case (black curve). The instability bound is represented by the red-shaded area.

By increasing the value of y_ν between 0.1 and 1 (in steps of 0.1), the instability scale dangerously approaches the heavy neutrino threshold (see Fig. 9), and in order to comply with the existence of the EW vacuum the model must be UV completed before entering the instability region. Using the approximate expression for $\Delta_{\text{approx}}^{\text{BSM}} \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} - 1$ in Eq. (4.5) of [94] we obtain that $y_\nu = 0.8$ corresponds to $\Delta_{\text{approx}}^{\text{BSM}} = 0.1$ %. Hence, from Fig. 9 we read that modifications of the trilinear Higgs self-coupling above the per mil level require an UV completion within a few orders of magnitude from the scale where the heavy neutrinos are integrated in.

4 Conclusions

In this paper we have addressed the question on how much could the trilinear Higgs self-coupling deviate from its SM value. We first discussed in Sect. 2 theoretical constraints on Higgs self-couplings from a general standpoint by considering two main arguments: vacuum instability and perturbativity. We showed that the former cannot be reliably assessed in a model-independent way, due to the breakdown of the EFT in describing the global structure of the Higgs potential away from the EW minimum. In particular, we have explicitly shown that by augmenting the SM via an $|H|^6$ operator one can generate two type of instabilities, either at large field values $v \ll H \lesssim \Lambda$ or in $H = 0$. In both cases, however, any reliable statement about the stability of the EW vacuum entails the knowledge of the full tower of effective operators, thus jeopardizing the connection with the Higgs self-couplings, whose leading order deviations are still governed by the $d = 6$ operators.

On the other hand, it is possible to use perturbativity in order to set fairly model-independent limits on Higgs self-couplings. In Sect. 2.3 we have employed two different criteria, based either on the partial-wave unitarity of the $hh \rightarrow hh$ scattering or on the loop corrections of the tree-level vertices, in order to establish the perturbative domain of the Higgs self-couplings. Though being conceptually different, the two criteria agree well with each other (cf. Eq. (2.28) with Eqs. (2.34)–(2.35)). Let us stress that indirect tests of the trilinear Higgs self-coupling either via single Higgs production [31–33] or EW precision tests [28, 29] and current measurements of non-resonant Higgs pair production [12] bound values of λ_{hhh} which are, at the moment, well above our perturbativity limit $|\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}| \lesssim 6$.

In the second part of the paper (Sect. 3), we investigated the size of the trilinear Higgs self-coupling in explicit models. First, we identified the class of models potentially leading to the largest modifications in the trilinear Higgs self-coupling, namely scalar extensions featuring a tadpole operator of the type $\mathcal{O}_\Phi = \Phi f(H)$, where $f(H)$ is a string of Higgs fields. The list of new scalars coupling linearly to H can be found in Table 1. They include both custodial symmetric (EW singlet and doublet) and custodial violating (EW triplets and quadruplets) scalar extensions. As two representative examples, we studied in detail the size of the trilinear Higgs self-coupling in the singlet and triplet extension, by taking into account constraints from EW precision tests, Higgs coupling measurements, direct searches for new scalars, vacuum stability and perturbativity. While in the singlet case modifications of the trilinear Higgs self-coupling up to factors of a few (close to the perturbativity limit) are still possible, for the triplet extension only modifications of a few percent can be expected.

Remarkably, vacuum stability is not a crucial discriminant for limiting the size of the trilinear Higgs self-coupling in models featuring new scalars, where the intricate structure of the scalar potential allows for regions in parameter space where large quartics (at the boundary of perturbativity) can tame the instabilities triggered by the tadpole operators. On the other hand, we have also found circumstances where vacuum stability can be very relevant. That is the case in which the trilinear Higgs self-coupling is modified by loops of heavy fermions. In our explicit example in Sect. 3.3 we have considered the case of low-scale

seesaw models, where the vacuum metastability bound can sizably reduce the allowed range for the trilinear Higgs self-coupling.

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A Parameters’ definitions

In this appendix we collect some details on the scalar potential (e.g. tadpole equations and scalar spectrum) for the two models studied in Sect. 3.

A.1 Singlet

The scalar fields can be expanded around their vevs by

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h \end{pmatrix}, \quad \Phi = (v_S + S), \quad (\text{A.1})$$

where we employed the unitary gauge for the Higgs doublet. The tadpole conditions can be written as

$$-\mu_4 v_S - \frac{\lambda_3 v_S^2}{2} - \mu_1^2 - \lambda_1 v_H^2 = 0, \quad (\text{A.2})$$

$$-\frac{\mu_4 v_H^2}{2} - \frac{1}{2} \lambda_3 v_H^2 v_S - \mu_2^2 v_S - \lambda_2 v_S^3 - \mu_3 v_S^2 = 0. \quad (\text{A.3})$$

The first condition allows to replace μ_1^2 in terms of v_H . The mass matrix in the real (h, S) basis then reads

$$\mathcal{M}_0^2 = \begin{pmatrix} m_{hh} & m_{hS} \\ m_{hS} & m_{SS} \end{pmatrix}, \quad (\text{A.4})$$

with

$$m_{hh} = 2v_H^2 \lambda_1, \quad (\text{A.5})$$

$$m_{hS} = v_H (\mu_4 + \lambda_3 v_S), \quad (\text{A.6})$$

$$m_{SS} = \mu_2^2 + \frac{1}{2} (\lambda_3 v_H^2 + 6v_S^2 \lambda_2 + 4v_S \mu_3). \quad (\text{A.7})$$

The mass matrix is diagonalized by rotating

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}, \quad (\text{A.8})$$

with

$$\tan 2\theta = \frac{2m_{hs}}{m_{ss} - m_{hh}}, \quad (\text{A.9})$$

and mass eigenvalues

$$\begin{aligned} m_{1,2}^2 &= \frac{1}{2} \left(m_{hh} + m_{ss} \mp \sqrt{4m_{hs}^2 + (m_{hh} - m_{ss})^2} \right) \\ &= \frac{1}{2} \left(m_{hh} + m_{ss} \pm (m_{hh} - m_{ss}) \frac{1}{\cos 2\theta} \right). \end{aligned} \quad (\text{A.10})$$

Expressing the couplings of the potential in terms of the parameters used for the scan, we find

$$\mu_1^2 = -\frac{1}{4} \left[(-2\lambda_3 v_s^2 + m_1^2 + m_2^2) + \cos(2\theta)(m_1^2 - m_2^2) - 2\frac{v_s}{v_H} \sin(2\theta)(m_1^2 - m_2^2) \right], \quad (\text{A.11})$$

$$\mu_2^2 = \frac{1}{2} \left[(\lambda_3 v_H^2 - m_1^2 - m_2^2 + 2\lambda_2 v_s^2) + \frac{v_H}{v_s} \sin(2\theta)(m_1^2 - m_2^2) + \cos(2\theta)(m_1^2 - m_2^2) \right], \quad (\text{A.12})$$

$$\mu_3 = \frac{1}{2v_s} \left[(m_1^2 + m_2^2 - \lambda_3 v_H^2 - 4\lambda_2 v_s^2) - \frac{1}{2} \frac{v_H}{v_s} \sin(2\theta)(m_1^2 - m_2^2) - \cos(2\theta)(m_1^2 - m_2^2) \right], \quad (\text{A.13})$$

$$\mu_4 = \frac{\sin(2\theta)(m_2^2 - m_1^2) - 2\lambda_3 v_H v_s}{2v_H}, \quad (\text{A.14})$$

$$\lambda_1 = \frac{\cos(2\theta)(m_1^2 - m_2^2) + m_1^2 + m_2^2}{4v_H^2}. \quad (\text{A.15})$$

$$(\text{A.16})$$

A.2 Triplet

The scalar fields can be expanded around their EM-conserving vevs via

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} (v_H + h^0 + iG^0) \end{pmatrix}, \quad \Phi = \begin{pmatrix} \eta_1 \\ \eta_2 \\ v_T + \eta_0 \end{pmatrix}, \quad (\text{A.17})$$

where the charged eigenstates for Φ are defined as $\eta^\pm = \frac{1}{\sqrt{2}} (\eta_1 \mp i\eta_2)$. Then the tadpole conditions can be written as

$$0 = \mu_1^2 + \lambda_1 v_H^2 + \frac{\lambda_3}{2} v_T^2 - \mu_4 v_T, \quad (\text{A.18})$$

$$0 = v_T \left(\mu_2^2 + \lambda_2 v_T^2 + \frac{\lambda_3}{2} v_H^2 \right) - \frac{\mu_4}{2} v_H^2. \quad (\text{A.19})$$

For $v_T = 0$ there is no doublet/triplet mixing and Eq. (A.19) implies $\mu_4 = 0$, which corresponds to the custodial symmetric tree-level relation $\rho = 1$. From now on we will assume $v_T \neq 0$. By evaluating the second derivatives of the scalar potential and after imposing the stationary Eqs. (A.18)–(A.19), we find the following scalar spectrum:

- Charged scalars: in the complex (ϕ^+, η^+) basis

$$\mathcal{M}_+^2 = \begin{pmatrix} 2\mu_4 v_T & \mu_4 v_H \\ \mu_4 v_H & \frac{\mu_4 v_H^2}{2v_T} \end{pmatrix}, \quad (\text{A.20})$$

which features a null eigenvalue, corresponding to the Goldstone boson G^+ eaten by the W and a massive state h^\pm with mass

$$m_{h^\pm}^2 = \frac{\mu_4 (v_H^2 + 4v_T^2)}{2v_T}. \quad (\text{A.21})$$

- Neutral pseudo-scalar: G^0 , corresponding to the Goldstone boson eaten by the Z .
- Neutral scalars: in the real (h^0, η^0) basis

$$\mathcal{M}_0^2 = \begin{pmatrix} m_{hh} & m_{h\eta} \\ m_{h\eta} & m_{\eta\eta} \end{pmatrix}, \quad (\text{A.22})$$

with

$$m_{hh} = 2\lambda_1 v_H^2, \quad (\text{A.23})$$

$$m_{h\eta} = v (\lambda_3 v_T - \mu_4), \quad (\text{A.24})$$

$$m_{\eta\eta} = 2\lambda_2 v_T^2 + \frac{\mu_4 v_H^2}{2v_T}. \quad (\text{A.25})$$

The mass eigenstates are obtained via the rotation

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix}, \quad (\text{A.26})$$

with

$$\tan 2\theta = \frac{2v_H (\lambda_3 v_T - \mu_4)}{2\lambda_2 v_T^2 + \frac{\mu_4 v_H^2}{2v_T} - 2\lambda_1 v_H^2}, \quad (\text{A.27})$$

and mass eigenvalues

$$\begin{aligned} m_{1,2}^2 &= \frac{1}{2} \left(m_{hh} + m_{\eta\eta} \mp \sqrt{4m_{h\eta}^2 + (m_{hh} - m_{\eta\eta})^2} \right) \\ &= \frac{1}{2} \left(m_{hh} + m_{\eta\eta} \pm (m_{hh} - m_{\eta\eta}) \frac{1}{\cos 2\theta} \right). \end{aligned} \quad (\text{A.28})$$

Moreover, the W boson mass is given by

$$m_W^2 = \frac{g^2}{4} (v_H^2 + 4v_T^2), \quad (\text{A.29})$$

which fixes $v^2 = (246.2 \text{ GeV})^2 = v_H^2 + 4v_T^2$, while EW precision tests set a bound on the custodial-breaking vev $v_T \lesssim 3.5 \text{ GeV}$. Summarising, an independent set of parameters can be chosen as:

$$v_H = \sqrt{v^2 - 4v_T^2}, \quad v_T < 3.55 \text{ GeV}, \quad m_1 = 125 \text{ GeV}, \quad m_2, \quad m_{h^\pm}, \quad \theta. \quad (\text{A.30})$$

Note however that instead of m_2 we scan over λ_2 . For completeness, we report here the potential parameters expressed in terms of those in Eq. (A.30)

$$\mu_4 = \frac{2m_{h^\pm}^2 v_T}{v_H^2 + 4v_T^2}, \quad (\text{A.31})$$

$$\lambda_1 = \frac{m_1^2 + m_2^2 + (m_1^2 - m_2^2) \cos 2\theta}{4v_H^2}, \quad (\text{A.32})$$

$$\lambda_2 = \frac{(m_1^2 + m_2^2)v_T - \mu_4 v_H^2 + (m_2^2 - m_1^2)v_T \cos 2\theta}{4v_T^3}, \quad (\text{A.33})$$

$$\lambda_3 = \frac{2\mu_4 v_H + (m_2^2 - m_1^2) \sin 2\theta}{2v_H v_T}. \quad (\text{A.34})$$

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